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Static, non-SUSY p-branes in diverse dimensions

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Abstract: We give explicit constructions of static, non-supersymmetric p-brane (for $p \leq d-4$, where d is the space-time dimensionality and including $p = -1$ or D-instanton) solutions of type II supergravities in diverse dimensions. A subclass of these are the static counterpart of the time dependent solutions obtained in [3]. Depending on the forms of the non-extremality function $G(r)$ defined in the text, we discuss various possible solutions and their region of validity. We show how one class of these solutions interpolate between the p-brane–anti-p-brane solutions and the usual BPS p-brane solutions in $d = 10$, while the other class, although have BPS limits, do not have such an interpretation. We point out how the time dependent solutions mentioned above can be obtained by a Wick rotation of one class of these static solutions. We also discuss another type of solutions which might seem non-supersymmetric, but we show by a coordinate transformation that they are nothing but the near horizon limits of the various BPS p-branes already known.

Keywords: Superstrings and Heterotic Strings, M-Theory, p-branes, D-branes.

Contents

1. Introduction

Type-II supergravities admit static, supersymmetric space-time geometries with isometries ISO$(p,\,1) \times$ SO$(d-p-1)$ in d-dimensions known as BPS p-branes [1, 2]. If one wants to construct an analogous time-dependent geometries with isometries ISO$(p+1) \times$ SO$(d-p-2,\,1)$, one finds that there are no real euclidean p-brane (or S-brane) solutions in this case [3]. However, if we do not insist on supersymmetries, then there exist real solutions with metrics having the aforementioned isometries [3]. So, it would be natural to ask whether there is an analogous static, non-supersymmetric p-brane solutions with isometries ISO$(p,\,1) \times$ SO$(d-p-1)$ in type-II supergravities in arbitrary d space-time dimensions and we find the answer in the positive. These solutions are not of the type of black p-branes [1] which are also non-supersymmetric but have isometries R $\times$ ISO$(p) \times$ SO$(d-p-1)$. We give explicit constructions of these solutions. Although a subclass of these solutions were previously known [4, 5, 6], in a different form, we give explicit constructions to facilitate our discussion on certain aspects of these solutions not considered before.

We construct the solutions by solving the equations of motion of type-II supergravities in d space-time dimensions containing a graviton, a dilaton and a $(d-p-3)$-form gauge field. It is well-known that when the supersymmetry condition is imposed the equations of motion lead to the usual BPS p-brane solutions [1, 2, 7]. However, in analogy with the time-dependent solutions [3] we relax the supersymmetry condition by introducing a non-extremality function $G(r)$ (defined below) and find a real magnetically (electrically) charged p-brane (for $-1 \leq p \leq d-4$) solutions which are characterized by three or less number of parameters. For the consistency of the equations of motion, we find that the non-extremality function can not be arbitrary and should take some specific forms. Demanding the asymptotic flatness of the metric we find that the non-extremality function $G(r)$ can be of the forms $G_{\mp}(r) = 1 \mp \omega^{2(d-p-3)}/r^{2(d-p-3)}$, where ω is a real integration constant. The upper sign leads to the three or two parameter static, non-supersymmetric p-brane solutions, whereas the lower sign leads to only two parameter solutions. Usually these solutions have singularities and we discuss the region of validity for these solutions. Since $p = -1$ or the case of D-instanton is quite different from the rest of the p-brane solutions we discuss it separately. Then we clarify the relations between the three parameter solutions and those obtained in [4]. Next we show, restricting our discussion in $d = 10$ that when the non-extremality function has the upper sign, the three parameter solutions nicely interpolate between the p-brane–anti-p-brane system with zero net p-brane charge and the usual BPS p-branes by scaling the parameters of the solutions in two distinct ways for $0 \leq p \leq 6$ and in a unique way for $p = -1$. This does not happen for the solutions with the non-extremality function having the lower sign. However, we find that even in this case it is possible to obtain the BPS solutions by appropriately scaling the parameters for $0 \leq p \leq 6$ but not for $p = -1$ or D-instanton. These solutions are not of the type of the usual BPS p-branes. It should be emphasized that here we consider these solutions as just the classical supergravity solutions and will not try to give any microscopic string interpretation. One such possible interpretation was given in [9] by considering these solutions (actually a subclass of these solutions (the three parameter solutions) with the non-extremality function having the upper sign) in $d = 10$ as the coincident Dp-$\bar{\text{D}}p$ branes and the three parameters of these solutions were argued to be related to the numbers N of Dp-branes, $\bar{N}$ of $\bar{\text{D}}p$-branes and the tachyon vev (T) of the brane-antibrane system. Recently we proposed [10] an exact relationships of the parameters of these solutions to the physically relevant parameters N, $\bar{N}$ and T and have shown how these relations are consistent with the right picture of tachyon condensation [11] on the brane-antibrane system. Time-dependent solutions can sometimes be obtained from the static solutions by applying Wick rotation. While it is known that the Wick rotations of the static BPS p-brane solutions do not lead to real time-dependent solutions, we show how a subclass of these non-supersymmetric solutions lead to the real time-dependent solutions obtained in [3]. Finally, we also discuss another type of solutions where we do not demand the asymptotic flatness of the metric and take the non-extremality function to be of the form $G(r) = \omega^{2(d-p-3)}/r^{2(d-p-3)}$. Although these solutions apparently seem to be non-supersymmetric, but actually in $d = 10$ they can be shown by a coordinate transformation to be the near-horizon limits of various BPS p-branes we know [12, 13].

This paper is organized as follows. In section 2, we describe the construction of static, non-supersymmetric p-brane solutions in d-dimensional supergravities. Various aspects of these solutions are discussed in section 3. In section 4, we obtain another class of solutions which are shown to be the near horizon limits of various BPS p-branes by a coordinate transformation. Our conclusion is presented in section 5.

2. General static, non-SUSY p-branes

In this section we describe the construction of static, non-supersymmetric p-branes by solving the equations of motion of type-II supergravities in d dimensions. The d-dimensional supergravity action containing a metric, a dilaton and a $(q-1) = (d-p-3)$-form gauge

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field with dilaton coupling a has the form,

$$S = \int d^d x \sqrt{-g} \left[R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2 \cdot q!} e^{a\phi} F_{[q]}^2 \right] . \tag{2.1}$$

The above action is quite general and consists of the bosonic sector of (dimensionally reduced) string/M theories. We emphasize here that the action (2.1) represents the low energy effective action of a critical string/M theories dimensionally reduced to d space-time dimensions and should not be considered as the corresponding action of a d-dimensional non-critical string theory. The equations of motion following from (2.1) are,

$$R_{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - \frac{e^{a\phi}}{2(q-1)!} \left[F_{\mu\alpha_2 \ldots \alpha_q} F_\nu{}^{\alpha_2 \ldots \alpha_q} - \frac{q-1}{q(d-2)} F_{[q]}^2 g_{\mu\nu} \right] = 0 \tag{2.2}$$

$$\partial_\mu \left(\sqrt{-g} e^{a\phi} F^{\mu\alpha_2 \ldots \alpha_q} \right) = 0 \tag{2.3}$$

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \partial^\mu \phi \right) - \frac{a}{2 \cdot q!} e^{a\phi} F_{[q]}^2 = 0 . \tag{2.4}$$

We will solve the equations of motion with the following ansatz,

$$ds^2 = e^{2A(r)} \left(dr^2 + r^2 d\Omega_{d-p-2}^2 \right) + e^{2B(r)} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2 \right) \tag{2.5}$$

$$F_{[q]} = b \, \mathrm{Vol}(\Omega_{d-p-2}) \tag{2.6}$$

In the above $r = (x_{p+1}^2 + \cdots + x_{d-1}^2)^{1/2}$, $d\Omega_{d-p-2}^2$ is the line element of a unit $(d-p-2)$-dimensional sphere, $\mathrm{Vol}(\Omega_{d-p-2})$ is its volume-form and b is the magnetic charge parameter. The space-time in (2.5) has the isometry $\mathrm{SO}(d-p-1) \times \mathrm{ISO}(p,1)$ and therefore the above represent a magnetically charged p-brane in d dimensions. Note here that since d is arbitrary, for $d = 11$, we have non-supersymmetric M-branes (with appropriate field contents) and when $d = 10$ we have non-supersymmetric p-branes of type-II string theories. However, when $d < 10$, the p-branes can be regarded as wrapped or delocalized branes (in analogy with the usual BPS p-branes) if we try to interpret them as full M/string theory branes.[1] It is well known that the solution is supersymmetric saturating the BPS bound if the function $A(r)$ and $B(r)$ satisfy

$$(p+1)B(r) + (q-1)A(r) = 0 . \tag{2.7}$$

Actually the condition of the preservation of some fraction of supersymmetries gets translated to the above condition on the metric and was shown in [7], with $p + 1 = d$ and $q - 1 = \tilde{d}$ in their notation. It is well-known that the solutions of the equations of motion with (2.5)–(2.7) lead to the usual BPS p-branes [1, 2, 7]. We will relax the condition (2.7) by

$$(p+1)B(r) + (q-1)A(r) = \ln G(r) . \tag{2.8}$$

As long as $G(r) \neq 1$, we expect the solution to break all the space-time supersymmetries. However, we will give an example in the last section where $G(r) \neq 1$, still one can make a

[1] As mentioned before since the nonsupersymmetric branes interact with each other it is difficult to give full M/string theoretic interpretation of these branes even in $d = 11$ and $d = 10$.

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coordinate transformation and modify $A(r)$ accordingly such that the relation (2.8) reduces to the form (2.7) and the supersymmetry will be restored. This does not happen for the solutions considered in this section.

The non-vanishing Ricci tensor components can be obtained from (2.5) as,

$$R_{rr} = (p+1)\left[B'' + B'^2 - A'B'\right] + q\left[A'' + \frac{A'}{r}\right] \tag{2.9}$$

$$R_{xx} = -R_{tt} = e^{2B-2A}\left[B'' + (q-1)A'B' + (p+1)B'^2 + q\frac{B'}{r}\right] \tag{2.10}$$

$$R_{ab} = r^2\left[A'' + (q-1)A'^2 + (2q-1)\frac{A'}{r} + (p+1)B'\left(A' + \frac{1}{r}\right)\right]\bar{g}_{ab} \tag{2.11}$$

where a, b are the indices for the transverse spherical (angular) coordinates and $\bar{g}_{ab}$ is the metric for the unit $q = (d-p-2)$-dimensional sphere. x denotes the indices for the longitudinal directions. Also 'prime' here denotes the derivative with respect to r. With the ansatz (2.6), eq. (2.3) is automatically satisfied. We rewrite the other equations of motion (2.2) and (2.4) using (2.9)–(2.11) and (2.8) as follows,

$$\begin{aligned} A'' + \frac{G''}{G} - \frac{G'^2}{G^2} + \frac{1}{p+1}\left(\frac{G'}{G} - (q-1)A'\right)^2 + (q-1)A'^2 - \frac{G'}{G}A' + \frac{q}{r}A' + \\ + \frac{1}{2}\phi'^2 - \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2q}} = 0 \end{aligned} \tag{2.12}$$

$$B'' + \frac{q}{r}B' + \frac{G'}{G}B' - \frac{b^2(q-1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2q}} = 0 \tag{2.13}$$

$$A'' + \frac{q}{r}A' + \frac{G'}{G}(A' + \frac{1}{r}) + \frac{b^2(p+1)}{2(d-2)}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2q}} = 0 \tag{2.14}$$

$$\phi'' + \frac{q}{r}\phi' + \frac{G'}{G}\phi' - \frac{ab^2}{2}\frac{e^{2(p+1)B+a\phi}}{G^2 r^{2q}} = 0\,. \tag{2.15}$$

Expressing eq. (2.14) in terms of $B(r)$ using (2.8) and substituting the equations of motion for $B(r)$ (eq. (2.13)), we get an equation involving the function $G(r)$ only as,

$$G'' + \frac{2q-1}{r}G' = 0\,. \tag{2.16}$$

Assuming $G(r)$ to go to unity asymptotically we find two solutions of the above equation as,

$$G_-(r) = 1 - \frac{\omega^{2(q-1)}}{r^{2(q-1)}}\,, \qquad G_+(r) = 1 + \frac{\tilde{\omega}^{2(q-1)}}{r^{2(q-1)}} \tag{2.17}$$

where both ω and $\tilde{\omega}$ are real. We factorize $G_-(r)$ and $G_+(r)$ as follows,

$$\begin{aligned} G_-(r) &= 1 - \frac{\omega^{2(q-1)}}{r^{2(q-1)}} = \left(1 + \frac{\omega^{q-1}}{r^{q-1}}\right)\left(1 - \frac{\omega^{q-1}}{r^{q-1}}\right) = H_1(r)\tilde{H}_1(r) \\ G_+(r) &= 1 + \frac{\tilde{\omega}^{2(q-1)}}{r^{2(q-1)}} = \left(1 + \frac{i\tilde{\omega}^{q-1}}{r^{q-1}}\right)\left(1 - \frac{i\tilde{\omega}^{q-1}}{r^{q-1}}\right) = H_2(r)\tilde{H}_2(r) \end{aligned} \tag{2.18}$$

where $H_1(r) = 1 + \omega^{q-1}/r^{q-1}$, $\tilde{H}_1(r) = 1 - \omega^{q-1}/r^{q-1}$, $H_2(r) = 1 + i\tilde{\omega}^{q-1}/r^{q-1}$ and $\tilde{H}_2(r) = 1 - i\tilde{\omega}^{q-1}/r^{q-1}$. We first solve the equations of motion with $G_-(r)$ having the form given in eq. (2.18). First using (2.13) and (2.15) we find

$$\left(\phi - \frac{a(d-2)}{q-1}B\right)'' + \frac{q}{r}\left(\phi - \frac{a(d-2)}{q-1}B\right)' + \frac{G'_-}{G_-}\left(\phi - \frac{a(d-2)}{q-1}B\right)' = 0\,. \tag{2.19}$$

The solution to this equation takes the form,

$$\phi = \frac{a(d-2)}{q-1}B + \delta \ln \frac{H_1}{\tilde{H}_1} \tag{2.20}$$

where δ is an arbitrary real constant. Now using (2.20) we express

$$e^{2(p+1)B + a\phi} = \left(\frac{H_1}{\tilde{H}_1}\right)^{a\delta} e^{B\chi} \tag{2.21}$$

where $\chi = 2(p+1) + a^2(d-2)/(q-1)$ and the equation for the function $B(r)$ in (2.13) takes the form,

$$B'' + \frac{q}{r}B' + \frac{G'_-}{G_-}B' - \frac{b^2(q-1)}{2(d-2)}\frac{e^{B\chi}H_1^{a\delta-2}}{r^{2q}\tilde{H}_1^{a\delta+2}} = 0\,. \tag{2.22}$$

Following the arguments given in ref. [3], we make the following ansatz for $B(r)$,

$$e^B = \left[\cosh^2\theta\left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha} - \sinh^2\theta\left(\frac{\tilde{H}_1}{H_1}\right)^{\beta}\right]^{\gamma} = F_1^{\gamma} \tag{2.23}$$

where $F_1 = [\cosh^2\theta(H_1/\tilde{H}_1)^{\alpha} - \sinh^2\theta(\tilde{H}_1/H_1)^{\beta}]$, with α, β, θ being some parameters and γ is another parameter which will be determined shortly. Note that the ansatz (2.23) differs from the similar ansatz (3.18) of ref. [3] for the time-dependent case. The reason we have hyperbolic function here instead of trigonometric function is that there is a sign difference in the last term of (2.22) from the corresponding equation in the time-dependent case. This will prove to be crucial to recover the BPS p-brane solution in this case (this was not possible for the time-dependent case as there is no BPS S-brane solution in type-II string theories) in the next section. Substituting (2.23) in (2.22) we obtain,

$$\left[\gamma(q-1)\omega^{2(q-1)}(\alpha+\beta)^2\sinh^2 2\theta\right]\frac{H_1^{\alpha-\beta}\tilde{H}_1^{\beta-\alpha}}{F_1^2} + \frac{b^2}{2(d-2)}H_1^{a\delta}\tilde{H}_1^{-a\delta}F_1^{\gamma\chi} = 0\,. \tag{2.24}$$

We thus obtain from here

$$\begin{aligned} \gamma\chi = -2\,, \qquad \alpha - \beta = a\delta \\ b = \sqrt{\frac{4(q-1)(d-2)}{\chi}}(\alpha+\beta)\omega^{q-1}\sinh 2\theta\,. \end{aligned} \tag{2.25}$$

We have taken $\alpha + \beta$, b and $\theta \geq 0$ without any loss of generality. From (2.25) we note that the parameter γ gets fixed and among α, β, δ only two are independent. However the consistency of the equations of motion (2.12) gives a relation between the parameter α and δ as,

$$\frac{1}{2}\delta^2 + \frac{2\alpha(\alpha - a\delta)(d-2)}{\chi(q-1)} = \frac{q}{q-1}\,. \tag{2.26}$$

From (2.26) we can determine both α and β in terms of δ as,

$$\begin{aligned} \alpha &= \sqrt{\frac{\chi q}{2(d-2)} - \frac{\delta^2}{4}\left(\frac{\chi(q-1)}{d-2} - a^2\right)} + \frac{a\delta}{2} \\ \beta &= \sqrt{\frac{\chi q}{2(d-2)} - \frac{\delta^2}{4}\left(\frac{\chi(q-1)}{d-2} - a^2\right)} - \frac{a\delta}{2} \end{aligned} \tag{2.27}$$

Note in the above that even though δ is real, the parameters α and β are not necessarily real. In fact depending on the value of δ we have two cases,

$$\begin{aligned} &(i)\ |\delta| \leq \sqrt{\frac{\chi q}{(q-1)(p+1)}}, \quad \text{then } \alpha\,,\ \beta \text{ are both real} \\ &(ii)\ |\delta| > \sqrt{\frac{\chi q}{(q-1)(p+1)}}, \quad \text{then } \alpha\,,\ \beta \text{ are both complex} \end{aligned} \tag{2.28}$$

For case (i) $|\delta|$ is bounded, on the other hand for case (ii) $|\delta|$ can be arbitrarily large. We thus obtain from (2.25) and (2.8)

$$\begin{aligned} e^{2B} &= F_1^{-\frac{4}{\chi}} \\ e^{2A} &= (H_1\tilde{H}_1)^{\frac{2}{q-1}} F_1^{\frac{4(p+1)}{(q-1)\chi}} \end{aligned} \tag{2.29}$$

and the complete non-supersymmetric, static, magnetically charged p-brane solutions as,

$$\begin{aligned} ds^2 &= F_1^{\frac{4(p+1)}{(q-1)\chi}} (H_1\tilde{H}_1)^{\frac{2}{q-1}} \left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + F_1^{-\frac{4}{\chi}} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= F_1^{-\frac{4a(d-2)}{(q-1)\chi}} \left(\frac{H_1}{\tilde{H}_1}\right)^{2\delta} \\ F_{[q]} &= b\ \mathrm{Vol}(\Omega_{d-p-2})\,. \end{aligned} \tag{2.30}$$

There are three independent paramaters δ, ω and θ characterizing the solutions (for case (i) above, see the discussion below). These solutions have some similarities with the BPS p-brane solutions in d dimensions. In fact, if H_1, $\tilde{H}_1 \to 1$ and $F_1 \to$ the usual harmonic function, then these solutions indeed reduce to the magnetically charged BPS p-brane solutions. We will come back to it in more detail in the next section.

The solutions (2.30) represent the magnetically charged p-branes. The corresponding electrically charged branes can be obtained from these solutions by using the transformation $g_{\mu\nu} \to g_{\mu\nu}$, $\phi \to -\phi$ and $F \to e^{-a\phi} * F$, where $*$ denotes the Hodge dual. So, for the electrically charged solutions the field strength can be calculated from above as,

$$F_{[p+2]} = e^{a\phi} * F_{[q]} \tag{2.31}$$

where the dilaton is as given in eq. (2.30). The $(p+1)$-form gauge field can be calculated from (2.31) as,

$$A_{[p+1]} = \sqrt{\frac{4(d-2)}{\chi(q-1)}} \sinh\theta \cosh\theta \left(\frac{C_1}{F_1}\right) dt \wedge \cdots \wedge dx_p \tag{2.32}$$

where,

$$C_1 = \left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha} - \left(\frac{\tilde{H}_1}{H_1}\right)^{\beta}. \tag{2.33}$$

Note that as long as $|\delta|$ is bounded as (i) in (2.28), α and β are real and so, F_1 given in (2.23) is also manifestly real and positive. But for case (ii) in (2.28) α and β are both complex and so, F_1 is not manifestly real. In this case let us write $\alpha = ic + a\delta/2$ and $\beta = ic - a\delta/2$ where $c = \sqrt{\frac{\delta^2}{4}(\frac{\chi(q-1)}{d-2} - a^2) - \frac{\chi q}{2(d-2)}}$ = positive.[2] Then we find from eq. (2.25) that since $(\alpha+\beta) = 2ic$ purely imaginary b will be real positive only for $\theta = -i\tilde{\theta}$. It can be easily checked that F_1 in this case will be real only for $\tilde{\theta} = \pi/4$ and takes the form, $F_1 = exp\{a\delta \tanh^{-1}(\omega^{q-1}/r^{q-1})\} \cos\left(2c \tanh^{-1}(\omega^{q-1}/r^{q-1})\right)$. With this F_1 the solution has the same form as in eq. (2.30). The gauge field in this case takes the form $A_{[p+1]} = \sqrt{\frac{4(d-2)}{\chi(q-1)}} \tan(2c \tanh^{-1}(\omega^{q-1}/r^{q-1})) dt \wedge \cdots \wedge dx_p$. Unlike in the previous case, the solutions now depend on two parameters ω and δ. These solutions are new and have not been considered before. They are quite unusual because of the presence of the periodic function and they are not well-defined everywhere in r and have possible singularities at $2c \tanh^{-1}(\omega^{q-1}/r^{q-1}) = n\pi + \pi/2$ with n an integer. These singularities are not enclosed by the corresponding event horizons, therefore naked. Our past experience [14, 15] tells that such singularities indicate the presence of an external source. We hope to understand the nature of the singularities and the associated issues better elsewhere. These solutions have actually very similar structure as the solutions obtained below with the non-extremality function $G_+(r)$ (whose singularity structure are discussed below eq. (2.41)) and so, we will not elaborate further on these solutions here.

Having described the solutions with the non-extremality function $G_-(r)$, we now discuss the solutions with the other non-extremality function $G_+(r)$ given in (2.18). It is clear that by letting ω^{q-1} purely imaginary i.e. $\omega^{q-1} \to i\tilde{\omega}^{q-1}$, $G_-(r) \to G_+(r)$. In other words the solutions with the non-extremality function $G_+(r)$ can be obtained from those with $G_-(r)$ by substituting $\omega^{q-1} = i\tilde{\omega}^{q-1}$, where $\tilde{\omega}^{q-1}$ is real. Now following the previous solution, we find from (2.20) that since the harmonic functions $H_2(r)$ and $\tilde{H}_2(r)$ are not real ($\ln(H_2/\tilde{H}_2) = 2i \tan^{-1}(\tilde{\omega}^{q-1}/r^{q-1})$, (purely imaginary)), so for the dilaton to remain real δ must be purely imaginary. Let us put $\delta = -i\tilde{\delta}$, where $\tilde{\delta}$ is real. Then the dilaton is given as,

$$\phi = \frac{a(d-2)}{q-1} B - i\tilde{\delta} \ln \frac{H_2}{\tilde{H}_2} = \frac{a(d-2)}{q-1} B + 2\tilde{\delta} \tan^{-1} \frac{\tilde{\omega}^{q-1}}{r^{q-1}}. \tag{2.34}$$

[2]One can show this, for example, by noting that $a^2 = 4 - \frac{2(p+1)(q-1)}{d-2}$ for supergravities with maximal susy in diverse dimensions [2].

Let us also put $\alpha = -i\tilde{\alpha}$ and $\beta = -i\tilde{\beta}$, however, $\tilde{\alpha}$ and $\tilde{\beta}$ are not real in general. If we substitute these in F_1 given by (2.23) it becomes,

$$F_1 \to F_2 = \cosh^2\theta e^{2\tilde{\alpha}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}} - \sinh^2\theta e^{-2\tilde{\beta}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}} . \tag{2.35}$$

Now substituting $e^B = F_2^\gamma$, in (2.22) we find for consistency

$$\gamma\chi = -2\,, \qquad \tilde{\alpha} - \tilde{\beta} = a\tilde{\delta}\,, \qquad b = \sqrt{\frac{4(q-1)(d-2)}{\chi}}(\tilde{\alpha} + \tilde{\beta})\tilde{\omega}^{q-1}\sinh 2\theta\,. \tag{2.36}$$

Also, the consistency of equation of motion (2.12) yields a relation among the parameters as,

$$\frac{1}{2}\tilde{\delta}^2 + \frac{2\tilde{\alpha}(\tilde{\alpha} - a\tilde{\delta})(d-2)}{\chi(q-1)} = -\frac{q}{q-1}\,. \tag{2.37}$$

From (2.37) we determine $\tilde{\alpha}$ and $\tilde{\beta}$ in terms of $\tilde{\delta}$ as,

$$\tilde{\alpha} = i\tilde{c} + \frac{a\tilde{\delta}}{2}\,, \qquad \tilde{\beta} = i\tilde{c} - \frac{a\tilde{\delta}}{2} \qquad \text{with } \tilde{c} = \sqrt{\frac{\chi q}{2(d-2)} + \frac{\tilde{\delta}^2}{4}\left(\frac{\chi(q-1)}{d-2} - a^2\right)}\,. \tag{2.38}$$

Note that $\tilde{c}$ in the above is real. Thus we find that both $\tilde{\alpha}$ and $\tilde{\beta}$ are complex. But $\tilde{\alpha} + \tilde{\beta} = 2i\tilde{c}$ is purely imaginary. Thus from the last relation of (2.36) we find that for b to remain real and positive θ must be purely imaginary i.e. $\theta = -i\tilde{\theta}$, where $\tilde{\theta}$ is real. Substituting this in F_2 (eq. (2.35)) we find that F_2 will remain real for $\tilde{\theta} = \pi/4$ only and in that csae F_2 becomes,

$$F_2 = e^{a\tilde{\delta}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}}\cos\left(2\tilde{c}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}\right) \tag{2.39}$$

The solutions in this case have precisely the same form as in eq. (2.30) with F_1 replaced by F_2 (given in eq. (2.39)) and H_1, $\tilde{H}_1$ replaced by H_2, $\tilde{H}_2$ and $\delta = -i\tilde{\delta}$. The complete solutions with $G_+(r)$ as the non-extremality function therefore are,

$$\begin{aligned}
ds^2 &= F_2^{\frac{4(p+1)}{(q-1)\chi}}\left(1 + \frac{\tilde{\omega}^{2(q-1)}}{r^{2(q-1)}}\right)^{\frac{2}{q-1}}\left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + F_2^{-\frac{4}{\chi}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right)\\
e^{2\phi} &= e^{\frac{8\tilde{\delta}(p+1)}{\chi}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}}\left[\cos\left(2\tilde{c}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}\right)\right]^{-\frac{4a(d-2)}{(q-1)\chi}}\\
F_{[q]} &= b\,\mathrm{Vol}(\Omega_{d-p-2})
\end{aligned} \tag{2.40}$$

For the electrical solutions, the gauge field can be obtained from (2.32) as,

$$A_{[p+1]} = \sqrt{\frac{4(d-2)}{\chi(q-1)}}\tan\left(2\tilde{c}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}\right)dt \wedge \cdots \wedge dx_p \tag{2.41}$$

We thus find that the solutions in this case are parameterized by only two parameters $\tilde{\omega}$ and $\tilde{\delta}$. Note that the parameter $\tilde{\delta}$ can be arbitrarily large as can be seen from eq. (2.38). This

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is important to obtain the BPS limits of these solutions and will be discussed in section 3. It should be noted that for both $G_-(r)$ and $G_+(r)$ in eq. (2.8), the p-brane solutions we obtain are not regular for all r between 0 and ∞. In fact for $G_-(r)$ with the case (i) given in (2.28), the three parameter solutions are not well-defined between $r=0$ and $r=\omega$ and at $r=0$, ω there are singularities. But for $r>\omega$, the solutions are regular. By definition, we need in general F_1 for $G_-(r)$ to be positive so that the corresponding configuration is well-defined.[3] This can be achieved in the above case if $r>\omega$. On the other hand for $G_-(r)$ with the case (ii) given in eq. (2.28), this is not enough because of the presence of the cosine function. We need in addition the range of validity for r to be outside of the following:

$$\frac{1+4n}{2}\pi \leq 2c\tanh^{-1}\frac{\omega^{q-1}}{r^{q-1}} \leq \frac{3+4n}{2}\pi\,, \qquad \text{for } n=0,1,2,\ldots \tag{2.42}$$

In other words the solutions for this case are well-defined if r satisfy the conditions

$$\begin{aligned} 0 &< 2c\tanh^{-1}\frac{\omega^{q-1}}{r^{q-1}} < \frac{\pi}{2} \\ \frac{3+4n}{2}\pi &< 2c\tanh^{-1}\frac{\omega^{q-1}}{r^{q-1}} < \frac{5+4n}{2}\pi\,, \qquad \text{for } n=0,1,2,\ldots \end{aligned} \tag{2.43}$$

Note that since $0<\tanh^{-1}\frac{\omega^{q-1}}{r^{q-1}}<\infty$, as $r>\omega$ and $c>0$, we therefore have $r>\omega/[\tanh(\pi/4c)]^{1/(q-1)}$ from the first equation of (2.43) and

$$\frac{\omega}{[\tanh(\frac{5+4n}{4c}\pi)]^{1/(q-1)}} < r < \frac{\omega}{[\tanh(\frac{3+4n}{4c}\pi)]^{1/(q-1)}} \tag{2.44}$$

from the second equation. The above implies that apart from the first case, the solutions are well-defined only in a finite region of r determined by c and integer n. For the case of $G_+(r)$ and to have a positive F_2, we find from eqs. (2.39) and (2.41), that r must lie outside the region given by (2.42) but with $c\tanh^{-1}\frac{\omega^{q-1}}{r^{q-1}}$ replaced by $\tilde{c}\tan^{-1}\frac{\tilde{\omega}^{q-1}}{r^{q-1}}$. We have similar replacement in eq. (2.43) for the present case. By the same reasoning as discussed in footnote 3, one can show now $2\tilde{c}>1$. Note also that by definition $0<\tan^{-1}(\tilde{\omega}^{q-1}/r^{q-1})<\pi/2$, since $r>0$. The analogous equation of the first one in (2.43) in the present case gives $r>\omega/[\tan(\pi/4\tilde{c})]^{1/(q-1)}$ and the remaining case is subtle and needs to be considered carefully as follows: (a) if $(3+4n)/2\tilde{c}>1$, then no solutions are allowed since the lower bound already exceeds $\pi/2$; (b) if $(3+4n)/2\tilde{c}<1$ but $(5+4n)/2\tilde{c}>1$ (i.e, $\tilde{c}/2-3/4>n>\tilde{c}/2-5/4$), the allowed region of validity for r is $\omega/[\tan(3+4n)\pi/(4\tilde{c})]^{1/(q-1)}>r>\omega/[\tan\pi/(4\tilde{c})]^{1/(q-1)}$; (c) if $(5+4n)/(2\tilde{c})<1$, the allowed region of validity is $\omega/[\tan(3+4n)\pi/(4\tilde{c})]^{1/(q-1)}>r>\omega/[\tan(5+4n)\pi/(4\tilde{c})]^{1/(q-1)}$. For each of the cases discussed above, a possible singularity can occur on the border of the region of validity for r. These singularities are naked in nature and therefore indicate the presence of an external source as mentioned earlier.

[3]There might exist some possibilities that this requirement can be relaxed. We will not discuss these in this paper.

2.1 $p = -1$-brane or D-instanton

So far we have discussed various static, non-supersymmetric p-brane solutions in type-II supergravities which in principle include $p = -1$ case. However, since the solutions are different in some respects we discuss this case separately. First, we note from the condition (2.5) that in this case there is no $B(r)$ in the metric. Also, since $q = d - p - 2 = d - 1$, so, from (2.8) we have

$$A(r) = \frac{1}{d-2} \ln G(r) \tag{2.45}$$

where as before $G(r)$ can take two forms

$$\begin{aligned} G_-(r) &= 1 - \frac{\omega^{2(d-2)}}{r^{2(d-2)}} = \left(1 + \frac{\omega^{d-2}}{r^{d-2}}\right)\left(1 - \frac{\omega^{d-2}}{r^{d-2}}\right) = H_1(r)\tilde{H}_1(r) \\ G_+(r) &= 1 + \frac{\tilde{\omega}^{2(d-2)}}{r^{2(d-2)}} = \left(1 + \frac{i\tilde{\omega}^{d-2}}{r^{d-2}}\right)\left(1 - \frac{i\tilde{\omega}^{d-2}}{r^{d-2}}\right) = H_2(r)\tilde{H}_2(r)\,. \end{aligned} \tag{2.46}$$

The above form of $A(r)$ is consistent with the equations of motion (2.14). The equation of motion (2.13) is absent and we rewrite the other two equations of motion (2.15) and (2.12) with $p = -1$, $q = d - 2$ as,

$$\phi'' + \frac{q}{r}\phi' + \frac{G'}{G}\phi' - \frac{ab^2}{2}\frac{e^{a\phi}}{G^2 r^{2(d-1)}} = 0 \tag{2.47}$$

$$(d-1)\left(A'' + \frac{A'}{r}\right) + \frac{1}{2}\phi'^2 - \frac{b^2}{2}\frac{e^{a\phi}}{G^2 r^{2(d-1)}} = 0\,. \tag{2.48}$$

As before the equation (2.47) can be solved for the non-extremality function $G_-(r)$ with an ansatz for e^{ϕ} as,

$$e^{\phi} = F_1^{\nu} \tag{2.49}$$

with

$$F_1 = \left[\cosh^2\theta\left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha} - \sinh^2\theta\left(\frac{\tilde{H}_1}{H_1}\right)^{\beta}\right], \qquad H_1 = 1 + \frac{\omega^{d-2}}{r^{d-2}}\,, \qquad \tilde{H}_1 = 1 - \frac{\omega^{d-2}}{r^{d-2}} \tag{2.50}$$

where α, β and θ are constant parameters. The parameter ν can be determined by substituting the above in (2.47) and we get,

$$\nu = -\frac{2}{a}\,, \quad \alpha = \beta\,, \quad \text{and} \quad b = \frac{4\alpha}{a}(d-2)\omega^{d-2}\sinh 2\theta\,. \tag{2.51}$$

Also, (2.48) determines the value of α as,

$$\alpha = a\sqrt{\frac{(d-1)}{2(d-2)}}\,. \tag{2.52}$$

So, the full non-supersymmetric D-instanton solution has the form,

$$\begin{aligned} ds^2 &= \left(H_1\tilde{H}_1\right)^{\frac{2}{d-2}}\left(dr^2 + r^2 d\Omega_{d-1}^2\right) \\ e^{2\phi} &= \left[\cosh^2\theta\left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha} - \sinh^2\theta\left(\frac{H_1}{\tilde{H}_1}\right)^{-\alpha}\right]^{-\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(\Omega_{d-1}) \end{aligned} \tag{2.53}$$

The scalar field for the corresponding electrically charged solution has the form

$$A_{[0]} = \frac{2i}{a} \sinh\theta \cosh\theta \left(\frac{C_1}{F_1}\right) \tag{2.54}$$

where $C_1 = (H_1/\tilde{H}_1)^{\alpha} - (H_1/\tilde{H}_1)^{-\alpha}$. Note here that the scalar $A_{[0]}$ is purely imaginary because we have used the same definition of Hodge duality as was used for the other p-brane solutions in (2.31). However, since for the instanton solution we need to go to euclidean coordinate or equivalently change the scalar $A_{[0]} = i\tilde{A}_{[0]}$ [16] (this changes the sign on the kinetic energy term of the scalar) and so for the instanton solution the scalar will take the form,

$$\tilde{A}_{[0]} = \frac{2}{a} \sinh\theta \cosh\theta \left(\frac{C_1}{F_1}\right). \tag{2.55}$$

The eqs. (2.53), (2.55) represent the D-instanton solution for the non-extremality function $G_-(r)$. Note that here the solution is charaterized by two parameters ω and θ in contrast to the three parameters for other values of p. Also since here α in (2.51) is real there is no solution analogous to the two parameter solution with $G_-(r)$ for the case of D-instanton. Now as before we will obtain the solution with the non-extremality function $G_+(r)$ by substituting in (2.53) and (2.55) the following

$$\omega^{d-2} \to i\tilde{\omega}^{d-2}\,, \qquad H_1,\, \tilde{H}_1 \to H_2,\, \tilde{H}_2\,, \quad \theta \to -i\tilde{\theta} = -\frac{i\pi}{4} \tag{2.56}$$

then the solutions take the forms,

$$\begin{aligned} ds^2 &= \left(1 + \frac{\tilde{\omega}^{2(d-2)}}{r^{2(d-2)}}\right)^{\frac{2}{d-2}} \left(dr^2 + r^2 d\Omega_{d-1}^2\right) \\ e^{2\phi} &= \left[\cos\left(2\alpha \tan^{-1}\frac{\tilde{\omega}^{d-2}}{r^{d-2}}\right)\right]^{-\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(\Omega_{d-1}) \end{aligned} \tag{2.57}$$

and for the electrically charged solution

$$A_{[0]} = \frac{2}{a} \tan\left(2\alpha \tan^{-1}\frac{\tilde{\omega}^{d-2}}{r^{d-2}}\right) \tag{2.58}$$

The singularity structures of the D-instanton solutions remain exactly the same as we have discussed earlier for other p-branes with $\tilde{c}$ replaced by α and so we will not repeat it here. We just mention that in $d = 10$ and for the usual dilaton coupling $a = (p-3)/2 = -2$, the instanton solution (2.57) and (2.58) is not well-defined as $r \to 0$ (given our previous discussion for $p \neq -1$, we know that r cannot be allowed to approach zero). A general non-supersymmetric D-instanton solution carrying electric charges of an SL(2,R) symmetry has been given recently in [18]. The singularity structure of these solutions and how to resolve them in some cases have been discussed there. In the next section, we will see, among other things, how BPS p-branes can be recovered from these non-supersymmetric p-branes by scaling the parameters in appropriate ways.

3. Discussion on some aspects of the solutions

In this section we will mainly discuss two aspects of the solutions obtained in the previous section, namely, how a subclass of the solutions can be regarded as the interpolating solutions between the p-brane–anti-p-brane system with zero net p-brane charge and the usual BPS p-branes[4] and then we point out how the Wick rotation of these solutions lead to the time-dependent solutions of ref. [3]. But before that we clarify the relations between the solutions obtained in section 2 and those given in ref. [4].

The solutions obtained by Zhou and Zhu in [4] are the generalized black p-branes in d dimensions and is given in eqs.(112)–(124) of their paper. The non-supersymmetric p-brane solutions in this paper correspond to $c_2 = 0$ of ref. [4]. Also we note that we should make the following identifications to compare the two solutions,

$$\begin{aligned} D &\equiv d\,, \qquad d \equiv p+1\,, \qquad \tilde{d} \equiv q-1\,,\\ \Delta &\equiv \frac{(q-1)\chi}{d-2}\,, \qquad r_0 \equiv \omega \end{aligned} \tag{3.1}$$

where in the above we have kept the symbols used by Zhou and Zhu on the left hand side and the symbols used in section 2 on the right. With these identifications, we find

$$h(r) = \ln \frac{\tilde{H}_1(r)}{H_1(r)}\,, \qquad \xi(r) = \ln H_1(r)\tilde{H}_1(r) = \ln G_-(r)\,. \tag{3.2}$$

We therefore have,

$$\cosh(\tilde{k}h(r)) + c_3 \sinh(\tilde{k}h(r)) = \frac{1}{2}(c_3+1)\left(\frac{H_1}{\tilde{H}_1}\right)^{-\tilde{k}} - \frac{1}{2}(c_3-1)\left(\frac{\tilde{H}_1}{H_1}\right)^{-\tilde{k}}. \tag{3.3}$$

Using these relations we simplify the metric and the prefactors multiplying the longitudinal as well as the transverse parts of the brane from eqs.(120) and (121) of ref. [4] and identify with $F_1^{-4/\chi}$ and $(H_1\tilde{H}_1)^{2/(q-1)}F_1^{4(p+1)/(q-1)\chi}$ respectively. We thus obtain,

$$F_1 \equiv \left(\frac{c_3+1}{2}\right)\left(\frac{H_1}{\tilde{H}_1}\right)^{\frac{ac_1}{2}-\tilde{k}} - \left(\frac{c_3-1}{2}\right)\left(\frac{\tilde{H}_1}{H_1}\right)^{-\frac{ac_1}{2}-\tilde{k}}. \tag{3.4}$$

Comparing with the form of F_1 in eq. (2.23), the parameters in the two solutions can be related as,

$$\begin{aligned} c_3 &= \cosh 2\theta\\ c_1 &= \delta\\ \tilde{k} &= -\frac{1}{2}(\alpha+\beta)\,. \end{aligned} \tag{3.5}$$

[4]In the context of open string tachyon condensation the non-BPS $(p+1)$-brane on the tachyonic kink goes over to a configuration which can be identified as BPS p-brane [8]. So, the case we are discussing here is not quite the same. However, the above process can be understood from a delocalized, non-supersymmetric p-brane solution and will be discussed elsewhere [17].

With (3.5), the parameter relation given in (117), (118) of ref. [4] reduce to eq. (2.26) i.e.,

$$-\frac{4\tilde{k}^2}{\Delta} = c_1^2 - \frac{a^2 c_1^2}{\Delta} - \frac{2(\tilde{d}+1)}{\tilde{d}} \quad \Rightarrow \quad \frac{1}{2}\delta^2 + \frac{2\alpha(\alpha - a\delta)(d-2)}{\chi(q-1)} = \frac{q}{q-1}\,. \tag{3.6}$$

We have thus clarified the relation of the solutions obtained in ref. [4] with the non-supersymmetric p-brane solutions we have obtained in eqs. (2.30), (2.32). We would like to point out that since ref. [4] contains only the three parameter solutions, we have clarified their relations with the three parameter solutions we have obtaned in section 2 with $G_-(r)$ as the non-extremality function. Also the D-instanton solution was not given in [4].

Let us now discuss (here we will restrict ourselves to $d = 10$) how we can regard the solutions (2.30), (2.32) as interpolating solutions between the Dp–anti-Dp system with net RR charge zero and the usual BPS Dp-branes. We will also discuss a similar interpretation as interpolating solution for the case of D-instanton solution (2.53), (2.55).[5] Note that like the Dp-antiDp system with net charge zero non-BPS Dp-branes also have net RR charge zero. The reason is the non-BPS Dp-branes of even (odd) dimensionalities exist in type-IIB (IIA) superstring theory as opposed to their BPS counterpart of odd (even) dimensionalities in the same theory [8]. However, we know that type-IIA (IIB) string theory contains odd (even) form RR gauge fields and the D-branes are charged under these gauge fields. Since a Dp-brane couples to a $(p+1)$-form gauge field, the charged Dp-branes in type-IIA (IIB) theory must be of even (odd) dimensionality. However, since the non-BPS Dp-branes are of opposite i.e. odd (even) dimensionalities in IIA (IIB) theories, they must be chargeless. So, what we want to emphasize here is that from an isolated supergravity solution it is not possible to distinguish a non-BPS Dp-brane from a Dp-antiDp system of zero RR charge [9, 19]. But since the general solutions (2.30), (2.32) interpolate (as we will show) between two solutions belonging to the same theory, so, if one solution is the BPS Dp-brane, the other one can not be non-BPS Dp-brane (since they do not belong to the same theory) and has to be coincident Dp-antiDp-brane system with zero net charge.

We note from the solutions (2.30) that $F_{[q]} = 0$ implies $b = 0$. Also from the third relation of (2.25) we find $b = 0$ implies $\theta = 0$. Therefore the function F_1 in (2.23) reduces to

$$F_1 = \left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha}. \tag{3.7}$$

The complete brane–anti-brane solutions can then be seen from (2.30), (2.32) to take the forms,

$$\begin{aligned} ds^2 &= (H_1\tilde{H}_1)^{\frac{2}{q-1}} \left(\frac{H_1}{\tilde{H}_1}\right)^{\frac{4(p+1)}{(q-1)\chi}\alpha} \left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + \left(\frac{H_1}{\tilde{H}_1}\right)^{-\frac{4}{\chi}\alpha} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= \left(\frac{H_1}{\tilde{H}_1}\right)^{-\frac{4a(d-2)}{(q-1)\chi}\alpha + 2\delta} \\ F_{[q]} &= 0\,, \qquad A_{[p+1]} = 0 \end{aligned} \tag{3.8}$$

[5] Note that this interpretation holds only for the three parameter solutions with the non-extremality function $G_-(r)$ and not for the two parameter solution with $G_-(r)$ and also for $G_+(r)$ (even though there exist BPS limits in these cases) and will be mentioned later.

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where the parameters α and δ are related by the first relation of eq. (2.27). These solutions are now characterized by two parameters ω and δ and in analogy with the arguments given in refs. [9, 10], the parameters would presumably be related to the mass and the tachyon vev of the underlying unstable brane–anti-brane system. In $d = 10$, $\chi = 32/(7-p)$, $a = (p-3)/2$, the solutions (3.8) simplify to,

$$
\begin{aligned}
ds^2 &= (H_1\tilde{H}_1)^{\frac{2}{7-p}} \left(\frac{H_1}{\tilde{H}_1}\right)^{\frac{(p+1)}{8}\alpha} (dr^2 + r^2 d\Omega_{8-p}^2) + \left(\frac{H_1}{\tilde{H}_1}\right)^{-\frac{7-p}{8}\alpha} (-dt^2 + dx_1^2 + \cdots + dx_p^2)\\
e^{2\phi} &= \left(\frac{H_1}{\tilde{H}_1}\right)^{\frac{3-p}{2}\alpha + 2\delta}\\
F_{[8-p]} &= 0\,, \qquad A_{[p+1]} = 0
\end{aligned}
\tag{3.9}
$$

with,

$$
\alpha = \sqrt{\frac{2(8-p)}{(7-p)} - \frac{(7-p)(p+1)}{16}\delta^2} + \frac{(p-3)\delta}{2}\,. \tag{3.10}
$$

This is exactly the same supergravity solutions obtained in refs. [9, 19] even though our interpretation here is different. For the case of D-instanton solution the corresponding brane–anti-brane solution can be obtained by setting as before $b = \theta = 0$ in (2.53), (2.55) and the solutions take the form,

$$
\begin{aligned}
ds^2 &= \left(H_1\tilde{H}_1\right)^{\frac{2}{d-2}} \left(dr^2 + r^2 d\Omega_{d-1}^2\right)\\
e^{2\phi} &= \left(\frac{H_1}{\tilde{H}_1}\right)^{-\frac{4\alpha}{a}}\\
F_{[d-1]} &= 0\,, \qquad A_{[0]} = 0
\end{aligned}
\tag{3.11}
$$

where $\alpha = a\sqrt{(d-1)/[2(d-2)]}$. In $d = 10$, they have the forms

$$
ds^2 = \left(H_1\tilde{H}_1\right)^{\frac{1}{4}} \left(dr^2 + r^2 d\Omega_9^2\right), \quad e^{2\phi} = \left(\frac{H_1}{\tilde{H}_1}\right)^{-3}, \quad F_{[9]} = 0\,, \quad A_{[0]} = 0 \tag{3.12}
$$

where $H_1 = 1 + \omega^8/r^8$, $\tilde{H}_1 = 1 - \omega^8/r^8$. We would like to point out that for the solutions with $G_+(r)$ as the non-extremality function, there are no non-trivial chargeless solutions. This is because in this case $\tilde{\theta} = \pi/4$ as we noted before and so, for the charge to vanish $\tilde{\omega}$ has to vanish for all p from -1 to 6 (see eq. (2.36)). So, the chargeless sloution in this case is trivial i.e. the flat space. (This conclusion also holds for the two parameter solutions with $G_-(r)$ as the non-extremality function.)

Now we will see how BPS p-branes can be obtained from the same solutions (2.30), (2.32). We first note from the third relation in (2.25) that if $(\alpha+\beta)\omega^{q-1} \to \epsilon\bar{\omega}^{q-1}$ and $\sinh 2\theta \to \epsilon^{-1}$, where $(\alpha+\beta) =$ finite, ϵ is a dimensionless parameter with $\epsilon \to 0$ and $\bar{\omega}^{q-1} = \sqrt{\frac{b^2\chi}{4(q-1)(d-2)}}$, then this relation reduces to the usual mass-charge relation of the magnetically charged BPS p-branes in d dimensions. (Note that $b =$ fixed in this case.)

Now it is clear that since $\omega^{q-1} \to \epsilon\,\bar{\omega}^{q-1}/(\alpha+\beta)$, both $H_1(r)$ and $\tilde{H}_1(r) \to 1$. On the other hand we find from (2.23)

$$
\begin{aligned}
F_1 &= \cosh^2\theta \left(\frac{H_1}{\tilde{H}_1}\right)^{\alpha} - \sinh^2\theta \left(\frac{\tilde{H}_1}{H_1}\right)^{\beta} \\
&= 1 + \frac{[(\alpha+\beta)\cosh 2\theta + (\alpha-\beta)]\,\omega^{q-1}}{r^{q-1}} \\
&\to 1 + \frac{\bar{\omega}^{q-1}}{r^{q-1}} = \bar{H}_1(r)\,.
\end{aligned}
\tag{3.13}
$$

Note here that the parameters α, β (or δ) remain arbitrary but they do not appear in the solutions. In these limits the gauge field in (2.32) reduces to

$$
A_{[p+1]} = \sqrt{\frac{4(d-2)}{\chi(q-1)}}\left(1 - \bar{H}_1^{-1}\right) dt \wedge dx_1 \wedge \cdots \wedge dx_p\,. \tag{3.14}
$$

We thus recover the BPS p-brane solutions in d dimensions from the solutions (2.30), (2.32) in the limit

$$
\begin{aligned}
(\alpha+\beta)\omega^{q-1} &\to \epsilon\,\bar{\omega}^{q-1} \\
\sinh 2\theta &\to \epsilon^{-1}
\end{aligned}
\tag{3.15}
$$

with $\epsilon \to 0$ and $(\alpha+\beta) =$ finite. In $d = 10$, $\chi = 32/(7-p)$ and using the above relations, the solutions (2.30), (2.32) take the forms,

$$
\begin{aligned}
ds^2 &= \bar{H}_1^{\frac{p+1}{8}}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) + \bar{H}_1^{-\frac{7-p}{8}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\
e^{2\phi} &= \bar{H}_1^{\frac{3-p}{2}} \\
F_{[8-p]} &= b\mathrm{Vol}(\Omega_{8-p})
\end{aligned}
\tag{3.16}
$$

for the magnetic brane and for the electric brane,

$$
A_{[p+1]} = \left(1 - \bar{H}_1^{-1}(r)\right) dt \wedge dx_1 \wedge \cdots \wedge dx_p\,. \tag{3.17}
$$

These are precisely the BPS magnetic and electric p-brane solutions in $d = 10$.

In deriving the BPS p-branes from the non-supersymmetric p-branes, we have taken the limits (3.15), where we also kept $\alpha+\beta =$ finite. However, we can also recover the BPS p-branes by taking another limit as,

$$
\begin{aligned}
\alpha+\beta &\to \epsilon^{\frac{1}{2}} \\
\omega^{q-1} &\to \epsilon^{\frac{1}{2}}\,\bar{\omega}^{q-1} \\
\sinh 2\theta &\to \epsilon^{-1}
\end{aligned}
\tag{3.18}
$$

where $\bar{\omega}$ is as defined before. We note that even in this case $H_1(r) \to 1$ and $\tilde{H}_1(r) \to 1$ and $F_1 \to \bar{H}_1(r) = 1 + \frac{\bar{\omega}^{q-1}}{r^{q-1}}$ as before and $A_{[p+1]} \to \sqrt{\frac{4(d-2)}{\chi(q-1)}}(1 - \bar{H}_1^{-1}(r))dt \wedge dx_1 \wedge \cdots \wedge dx_p =$ $(1 - \bar{H}_1^{-1}(r))dt \wedge dx_1 \wedge \cdots \wedge dx_p$ in $d = 10$. But unlike in the previous case where α, β (or

δ) remains arbitrary, here they scale. Since $\alpha + \beta = -2\tilde{k} = 2\sqrt{\frac{\chi q}{2(d-2)} - \frac{\delta^2}{4}(\frac{\chi(q-1)}{d-2} - a^2)}$, so, $\alpha + \beta \to \epsilon^{1/2}$ implies

$$|\delta| \to \left[\frac{\chi q}{(q-1)(p+1)}\right]^{\frac{1}{2}} - \frac{\epsilon\,(d-2)}{4[\chi q(q-1)(p+1)]^{\frac{1}{2}}}\,. \tag{3.19}$$

In $d = 10$, this limit has been taken in ref. [9] to recover BPS Dp-branes from the solutions given in [4].[6]

For the case of D-instanton solution (2.53), (2.55) a limit similar to (3.15) can be taken. However, since for this case α is fixed, there is no limit similar to (3.18). So, for $0 \leq p \leq 6$, there are two distinct ways in which BPS p-brane solutions can be recovered by scaling the parameters of the non-supersymmetric p-brane solutions, but for $p = -1$ or for D-instanton there is only one way the BPS solution can be recovered. Let us indicate how this is done for $p = -1$ case. We note from the expression of F_1 in (2.50) that with the following scaling of the parameters,

$$\begin{aligned} 2\alpha\omega^{d-2} &\to \epsilon\bar{\omega}^{d-2} \\ \sinh\theta &\to \epsilon^{-1} \end{aligned} \tag{3.20}$$

where $\epsilon \to 0$, F_1 reduces to,

$$F_1 \to \bar{H}_1 = 1 + \frac{\bar{\omega}^{d-2}}{r^{d-2}}\,. \tag{3.21}$$

Then the solutions (2.53), (2.55) reduce to

$$\begin{aligned} ds^2 &= \left(dr^2 + r^2 d\Omega_{d-1}^2\right) \\ e^{2\phi} &= (\bar{H}_1)^{-\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(\Omega_{d-1})\,, \qquad A_{[0]} = \frac{2}{a}\left(1 - \bar{H}_1^{-1}\right). \end{aligned} \tag{3.22}$$

In $d = 10$ and $a = -2$, this is precisely the D-instanton solution obtained in ref. [16]. This is a regular solution where the metric has the wormhole geometry in the string frame.

We have therefore shown how the non-supersymmetric p-brane solutions (2.30), (2.32) and (2.53), (2.55) can be regarded as interpolating solutions from brane–anti-brane solutions (for $\theta \to 0$) to BPS p-branes (for $\theta \to \infty$ and keeping b fixed). For the case of $0 \leq p \leq 6$, the BPS solutions were obtained in two different ways whereas for $p = -1$ it was obtained in only one way. We would also like to point out that in recovering BPS p-brane solutions $\sinh 2\theta$ has to go to infinity and this is not possible for trigonometric function which appears in the case of corresponding time-dependent solutions. This is consistent with the fact that for time-dependent case there are no real BPS solutions in type-II supergravities.

[6]Note here that the BPS limits we have discussed hold only for the solution when $|\delta|$ is bounded by case (i) of (2.28). However, for case (ii) there is also a BPS limit analogous to eq. (3.23) given below and the BPS solution in this case take the forms very similar to eq. (3.25). So, we do not elaborate the BPS limits in this case whose meanings are also not clear to us.

For the solutions (2.40) with $G_+(r)$ as the non-extremality function we mentioned before that there is no non-trivial chargeless solutions analogous to brane–anti-brane systems in this case. However, it is possible to obatin BPS solutions by scaling the parameters in appropriate ways. Let us indicate how this can be done from (2.40). We scale the parameters as follows,

$$\begin{aligned} \tilde{\omega}^{q-1} &\to \tilde{\epsilon}\bar{\omega}^{q-1} \\ \tilde{\delta} &\to \tilde{\epsilon}^{-1} \end{aligned} \tag{3.23}$$

where $\tilde{\epsilon}$ is a dimensionless parameter with $\tilde{\epsilon} \to 0$ and $\bar{\omega}^{q-1}$ = fixed. Note that with this scaling $G_+(r) \to 1$ and the condition (2.8) reduces to the supersymmetry condition. The function F_2 in (2.39) takes the form,

$$F_2 \to \bar{F}_2 = e^{a\frac{\bar{\omega}^{q-1}}{r^{q-1}}} \cos\left(2\bar{c}\frac{\bar{\omega}^{q-1}}{r^{q-1}}\right) \tag{3.24}$$

where $\bar{c} = \sqrt{\frac{(p+1)(q-1)}{2(d-2)}}$. It is clear that since F_2 contains a periodic function of r, it can not be reduced to the usual harmonic function of a BPS p-brane. The complete BPS solutions in this case have the forms,

$$\begin{aligned} ds^2 &= \bar{F}_2^{\frac{4(p+1)}{(q-1)\chi}}\left(dr^2 + r^2 d\Omega_{d-p-2}^2\right) + \bar{F}_2^{-\frac{4}{\chi}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= e^{4\frac{\bar{\omega}^{q-1}}{r^{q-1}}}\bar{F}_2^{-\frac{4a(d-2)}{(q-1)\chi}} \\ F_{[q]} &= b\ \mathrm{Vol}(\Omega_{d-p-2}) \end{aligned} \tag{3.25}$$

for the magnetically charged solutions and for the electrical solutions we have

$$A_{[p+1]} = \sqrt{\frac{4(d-2)}{\chi(q-1)}} \tan\left(2\bar{c}\frac{\bar{\omega}^{q-1}}{r^{q-1}}\right). \tag{3.26}$$

These BPS solutions are not of the usual BPS p-brane type as they involve periodic functions. Like the solutions (2.40) these are also not well-defined for all r between 0 and ∞. In fact these solutions are well-defined inside the range of r given by $r > \bar{\omega}/[\pi/4\bar{c}]^{1/(q-1)}$ or by

$$\frac{\bar{\omega}}{\left(\frac{5+4n}{2\bar{c}}\pi\right)^{\frac{1}{q-1}}} < r < \frac{\bar{\omega}}{\left(\frac{3+4n}{2\bar{c}}\pi\right)^{\frac{1}{q-1}}}, \qquad \text{for} \quad n = 0, 1, 2, \ldots \tag{3.27}$$

and $r = 0$ is excluded and there are singularities at

$$r = \frac{\bar{\omega}}{[\pi/4\bar{c}]^{1/(q-1)}}, \frac{\bar{\omega}}{\left(\frac{3+4n}{4\bar{c}}\pi\right)^{\frac{1}{q-1}}}, \frac{\bar{\omega}}{\left(\frac{5+4n}{4\bar{c}}\pi\right)^{\frac{1}{q-1}}}. \tag{3.28}$$

So, we mention that although the solutions (2.40) have BPS limits, these are quite unusual and therefore, (2.40) can not be interpreted as interpolating solutions of p-brane–anti-p-brane systems and the usual BPS p-branes as for the three parameter solutions with $G_-(r)$.

Next we show how by a Wick rotation on the static non-supersymmetric p-brane solutions given in (2.30), (2.32) we get the time-dependent solutions or space-like p-branes [20]–[24] (or Sp-branes) obtained in [3]. Usually the Wick rotations on the BPS p-branes do not lead to real solutions, however, in this case we get real solutions. Let us consider the solutions (2.30), (2.32) and apply the following Wick rotation,

$$\begin{aligned} r &\to it \\ t &\to ix_{p+1} \end{aligned} \tag{3.29}$$

along with $\omega \to i\omega$. We also write,

$$d\Omega^2_{d-p-2} = d\psi^2 + \sin^2\psi d\Omega^2_{d-p-3} \tag{3.30}$$

and make the Wick rotation $\psi \to -i\psi$. Under this change (3.29) reduces to,

$$d\Omega^2_{d-p-2} = d\psi^2 + \sin^2\psi d\Omega^2_{d-p-3} \to -d\psi^2 - \sinh^2\psi d\Omega^2_{d-p-3} = -dH^2_{d-p-2} \tag{3.31}$$

where dH^2_{d-p-2} is the line element for the $(d-p-2)$-dimensional hyperbolic space. By further changing $\theta \to i\theta$, we find that since

$$\begin{aligned} H_1(r) &= 1 + \frac{\omega^{q-1}}{r^{q-1}} \quad \to \quad 1 + \frac{\omega^{q-1}}{t^{q-1}} = H_1(t) \\ \tilde{H}_1(r) &= 1 - \frac{\omega^{q-1}}{r^{q-1}} \quad \to \quad 1 - \frac{\omega^{q-1}}{t^{q-1}} = \tilde{H}_1(t) \\ F_1(r) &= \cosh^2\theta \left(\frac{H_1(r)}{\tilde{H}_1(r)}\right)^{\alpha} - \sinh^2\theta \left(\frac{\tilde{H}_1(r)}{H_1(r)}\right)^{\beta} \\ &\to \cos^2\theta \left(\frac{H_1(t)}{\tilde{H}_1(t)}\right)^{\alpha} + \sin^2\theta \left(\frac{\tilde{H}_1(t)}{H_1(t)}\right)^{\beta} = F_1(t) \end{aligned} \tag{3.32}$$

the metric and the dilaton in (2.30) changes to,

$$\begin{aligned} ds^2 &= F_1(r)^{\frac{4(p+1)}{(q-1)\chi}} (H_1(r)\tilde{H}_1(r))^{\frac{2}{q-1}} \left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + F_1(r)^{-\frac{4}{\chi}}(-dt^2 + \cdots + dx_p^2) \\ &\to F_1(t)^{\frac{4(p+1)}{(q-1)\chi}} (H_1(t)\tilde{H}_1(t))^{\frac{2}{q-1}} \left(-dt^2 + t^2 dH^2_{d-p-2}\right) + F_1(t)^{-\frac{4}{\chi}}(dx_1^2 + \cdots + dx_{p+1}^2) \\ e^{2\phi} &= F_1(r)^{-\frac{4a(d-2)}{(q-1)\chi}} \left(\frac{H_1(r)}{\tilde{H}_1(r)}\right)^{2\delta} \quad \to \quad F_1(t)^{-\frac{4a(d-2)}{(q-1)\chi}} \left(\frac{H_1(t)}{\tilde{H}_1(t)}\right)^{2\delta}. \end{aligned} \tag{3.33}$$

Now in order to see how $F_{[q]}$ changes we first note that

$$\begin{aligned} \mathrm{Vol}(\Omega_{d-p-2}) &= (\sin\psi)^{d-p-3} d\psi \wedge \cdots \\ &\to (-i)^{d-p-2}(\sinh\psi)^{d-p-3} d\psi \wedge \cdots \\ &= (-i)^{d-p-2}\mathrm{Vol}(H_{d-p-2}) \end{aligned} \tag{3.34}$$

where $\mathrm{Vol}(H_{d-p-2})$ is the volume-form of the hyperbolic space. It is clear from (3.33) that in order to get a real solution b must change as $b \to (i)^{d-p-2}b$ and $F_{[q]}$ then changes to

$$F_{[q]} = b\mathrm{Vol}(\Omega_{d-p-2}) \to b\mathrm{Vol}(H_{d-p-2})\,. \tag{3.35}$$

It can be easily checked that the parameter relation changes as,

$$b = \sqrt{\frac{4(q-1)(d-2)}{\chi}}(\alpha+\beta)\omega^{q-1}\sinh 2\theta \quad \to \quad b = \sqrt{\frac{4(q-1)(d-2)}{\chi}}(\alpha+\beta)\omega^{q-1}\sin 2\theta\,. \tag{3.36}$$

With these changes the gauge field (2.32) changes to

$$A_{[p+1]} \to \sqrt{\frac{4(d-2)}{\chi(q-1)}}\sin\theta\cos\theta\left(\frac{C_1}{F_1}\right) dx_1\wedge\cdots\wedge dx_{p+1} \tag{3.37}$$

upto an overall sign. Eqs. (3.32), (3.34)–(3.36) are precisely the form of the time dependent solutions we obtained in [3].

Let us now look at the D-instanton solution (2.53) and (2.54)[7] with $G_-(r)$ as the non-extremality function. By using the same trick as applied for other p-branes, we here obtain the corresponding time-dependent solution or S(-1)-brane solution as,

$$\begin{aligned} ds^2 &= \left(1-\frac{\omega^{2(d-2)}}{t^{2(d-2)}}\right)^{\frac{2}{d-2}}\left(-dt^2+t^2dH_{d-1}^2\right) \\ e^{2\phi} &= \left[\cos^2\theta\left(\frac{H_1(t)}{\tilde{H}_1(t)}\right)^{\alpha}+\sin^2\theta\left(\frac{H_1(t)}{\tilde{H}_1(t)}\right)^{-\alpha}\right]^{-\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(H_{d-1})\,, \qquad A_{[0]} = \frac{2}{a}\sin\theta\cos\theta\left(\frac{C_1(t)}{F_1(t)}\right). \end{aligned} \tag{3.38}$$

In $d=10$ this is precisely the S(-1)-brane solution obtained in ref. [22]. It can be easily checked that for the p-brane solutions, eqs. (2.40), (2.41) including the D-instanton solution, eqs. (2.57), (2.58) with $G_+(r)$ as the non-extremality function, there are no real time-dependent solutions which can be obtained by Wick rotation. The reason is, in this case as we make the Wick rotation $r\to it$ and $\tilde{\omega}\to i\tilde{\omega}$, we note from (2.36) that b can not remain real, since $\theta = -i\tilde{\theta} = -i\pi/4$ and $\tilde{\alpha}+\tilde{\beta} = 2ic$, where c is real. Similar argument holds also for the two parameter solutions with $G_-(r)$ as the non-extremality function. Thus we can not get real time dependent solutions by Wick rotation in these cases and this is consistent with the observation made in ref. [3].

4. Another class of solutions

In this section we will discuss a different class of solutions of the equations of motion than those discussed in section 2. Here also we relax the supersymmetry condition and the function $A(r)$ and $B(r)$ appearing in the metric will be taken to satisfy (2.8). However, we will see that the solutions in this case can be reduced to supersymmetric solutions by a coordinate transformation. We will recognize the solutions to be the near horizon limits of the various BPS p-branes in $d=10$.

[7]The reason we are using (2.54) as the solution for the scalar instead of (2.55) is that for the case of S(-1)-brane we do not need to go to the euclidean coordinate as for the D(-1)-brane solution.

We have seen in section 2 that the equations of motion dictate that the function $G(r)$ defined in (2.8) must satisfy eq. (2.16). If we do not insist that $G(r)$ goes to unity asymptotically then the solution for $G(r)$ can take the form,

$$G(r) = \frac{\hat{\omega}^{2(q-1)}}{r^{2(q-1)}} = \hat{H}(r)^2 \tag{4.1}$$

where $\hat{H}(r) = \hat{\omega}^{q-1}/r^{q-1}$ is a harmonic function in the $(q+1)$-dimensional transverse space. Now comparing eqs. (2.13) and (2.15) we find,

$$\phi = \frac{a(d-2)}{q-1} B\,. \tag{4.2}$$

Let us now make an ansatz for B as

$$B(r) = \hat{\alpha} \ln \hat{H}(r) \tag{4.3}$$

where $\hat{\alpha}$ is a parameter to be determined from the equations of motion. Now substituting (4.2) and (4.3) into the equation for the function B in (2.13), we find that the equation simplifies to,

$$\hat{\alpha}(q-1)\hat{\omega}^{2(q-1)} = \frac{b^2}{2(d-2)} \frac{\hat{\omega}^{(\hat{\alpha}\chi-2)(q-1)}}{r^{(\hat{\alpha}\chi-2)(q-1)}}\,. \tag{4.4}$$

This equation can be solved if $\hat{\alpha}\chi = 2$ and the solution is

$$\hat{\omega}^{q-1} = \sqrt{\frac{b^2\chi}{4(q-1)(d-2)}}\,. \tag{4.5}$$

We note that this is exactly the form of $\bar{\omega}$ for the BPS Dp brane solutions we have defined earlier i.e., $\hat{\omega} = \bar{\omega}$. It can be easily checked that the R_{rr} equation (2.12) is automatically satisfied with this solution. We thus find,[8]

$$\begin{aligned} e^{2B} &= \hat{H}^{\frac{4}{\chi}} \\ e^{2A} &= \hat{H}^{-\frac{4(p+1)}{(q-1)\chi}+\frac{4}{q-1}}\,. \end{aligned} \tag{4.6}$$

The complete solution therefore takes the form,

$$\begin{aligned} ds^2 &= \hat{H}^{-\frac{4(p+1)}{(q-1)\chi}+\frac{4}{q-1}} \left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + \hat{H}^{\frac{4}{\chi}} \left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= \hat{H}^{\frac{4a(d-2)}{(q-1)\chi}} \\ F_{[q]} &= b\ \mathrm{Vol}(\Omega_{d-p-2}) \end{aligned} \tag{4.7}$$

and for the electric brane

$$A_{[p+1]} = \sqrt{\frac{4(d-2)}{(q-1)\chi}}\, \hat{H} dt \wedge dx_1 \wedge \cdots \wedge dx_p\,. \tag{4.8}$$

[8]We point out that the corresponding time-dependent solutions given in [3] has some typographical errors in eqs. (4.5)–(4.7). We here correct them by comparing with eqs. (4.6) and (4.7) given below.

One might be tempted to think that the solutions (4.7), (4.8) are non-supersymmetric as the functions $A(r)$ and $B(r)$ appearing in the metric do not satisfy the supersymmetry condition (2.7) rather they satify (2.8) where $G(r) = \left(\frac{\bar\omega^{q-1}}{r^{q-1}}\right)^2$. However, we show that by a coordinate transformation, the configurations given in (4.7) and (4.8) can be cast into the form of the near-horizon limits of various Dp-branes confirming that the solutions are indeed supersymmetric. We will mention also what happens to the condition (2.8) under this coordinate transformation. Let us write the configurations (4.7) and (4.8) as,

$$\begin{aligned} ds^2 &= \left(\frac{r}{\bar\omega}\right)^{\frac{4(p+1)}{\chi}} \frac{\bar\omega^4}{r^4}\left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) + \left(\frac{r}{\bar\omega}\right)^{-\frac{4(q-1)}{\chi}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= \left(\frac{r}{\bar\omega}\right)^{-\frac{4a(d-2)}{\chi}} \\ F_{[q]} &= b\ \mathrm{Vol}(\Omega_{d-p-2})\,, \qquad A_{[p+1]} = \sqrt{\frac{4(d-2)}{(q-1)\chi}}\frac{r^{-(q-1)}}{\bar\omega^{-(q-1)}} dt\wedge dx_1 \wedge \ldots \wedge dx_p\,. \end{aligned} \tag{4.9}$$

Now let us make a coordinate transformation

$$r = \frac{\bar\omega^2}{z} \tag{4.10}$$

Then we find

$$\frac{\bar\omega^4}{r^4}\left(dr^2 + r^2 d\Omega^2_{d-p-2}\right) = \left(dz^2 + z^2 d\Omega^2_{d-p-2}\right) \tag{4.11}$$

and we can rewrite (4.9) as,

$$\begin{aligned} ds^2 &= \bar H(z)^{\frac{4(p+1)}{(q-1)\chi}}\left(dz^2 + z^2 d\Omega^2_{d-p-2}\right) + \bar H(z)^{-\frac{4}{\chi}}\left(-dt^2 + dx_1^2 + \cdots + dx_p^2\right) \\ e^{2\phi} &= \bar H(z)^{-\frac{4a(d-2)}{(q-1)\chi}} \\ F_{[q]} &= b\ \mathrm{Vol}(\Omega_{d-p-2})\,, \qquad A_{[p+1]} = \sqrt{\frac{4(d-2)}{(q-1)\chi}}\bar H(z)^{-1} dt\wedge \cdots \wedge dx_p \end{aligned} \tag{4.12}$$

where $\bar H(z) = \frac{\bar\omega^{q-1}}{z^{q-1}}$ is the $z \to 0$ limit of the harmonic function we defined in section 3. We recognize [12, 13] (4.12) in $d = 10$ as the near-horizon limits of the BPS Dp-branes and so the solutions (4.12) are indeed supersymmetric. A similar solution can also be found for the case of D-instanton. As before there is no $B(r)$ in the metric and $A(r)$ would be given as,

$$A(r) = \frac{1}{d-2}\ln G(r) \tag{4.13}$$

where

$$G(r) = \frac{\hat\omega^{2(d-2)}}{r^{2(d-2)}} = \hat H^2(r)\,. \tag{4.14}$$

Now using the ansatz,

$$e^\phi = (\hat H(r))^{\hat\nu} \tag{4.15}$$

in eqs. (2.47) and (2.48) we obtain,

$$\hat\nu = \frac{2}{a}\,, \qquad b = \frac{2}{a}(d-2)\hat\omega^{d-2}\,. \tag{4.16}$$

The D-instanton solution therefore takes the form,

$$\begin{aligned} ds^2 &= \frac{\hat{\omega}^4}{r^4}\left(dr^2 + r^2 d\Omega_{d-1}^2\right) \\ e^{2\phi} &= (\hat{H}(r))^{\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(\Omega_{d-1})\,, \qquad A_{[0]} = \frac{2}{a}\hat{H}(r)\,. \end{aligned} \tag{4.17}$$

Again by making a coordinate transformation (4.10) we can rewrite the above configuration as,

$$\begin{aligned} ds^2 &= \left(dz^2 + z^2 d\Omega_{d-1}^2\right) \\ e^{2\phi} &= (\bar{H}(z))^{-\frac{4}{a}} \\ F_{[d-1]} &= b\mathrm{Vol}(\Omega_{d-1})\,, \qquad A_{[0]} = \frac{2}{a}\bar{H}(z)^{-1} \end{aligned} \tag{4.18}$$

where $\bar{H} = \bar{\omega}^{d-2}/z^{d-2}$ with $\bar{\omega} = \hat{\omega}$ and is the $z \to 0$ limit of the usual harmonic function. Eq. (4.18) is precisely the near horizon limit of the BPS D(-1)-brane solution in $d = 10$. We can also obtain the corresponding time-dependent solutions obtained in [3] by the Wick rotation discussed earlier.

Let us now try to understand what happens to the condition (2.8), i.e., $(p+1)B(r) + (q-1)A(r) = \ln G(r)$. Actually, when we make a change of variable from $r \to z$, $\ln G(r)$ gets absorbed into the function $A(r)$ i.e.,

$$\tilde{A}(z) = -A(r) + \frac{1}{q-1}\ln G(r) \tag{4.19}$$

where $e^{\tilde{A}(z)}$ is the factor multiplying the transverse part of the brane i.e., $(dz^2+z^2 d\Omega_{d-p-2}^2)$. Thus in terms of z-coordinate the relation (2.8) reduces to (2.7) i.e., supersymmetric condition. In other words,

$$(p+1)B(r) + (q-1)A(r) = \ln G(r) \quad \Rightarrow \quad (p+1)B(z) + (q-1)\tilde{A}(z) = 0\,. \tag{4.20}$$

This happens only for this particular form of the harmonic function $\hat{H}(r) = \frac{\bar{\omega}^{q-1}}{r^{q-1}}$ and $G(r) = \hat{H}(r)^2$. Thus we clarify the reason why starting from the non-supersymmetric gauge condition (2.8) we end up with supersymmetric solutions. Note that a similar solution in [3] for the time-dependent case were inappropriately called as the non-BPS Ep-branes in type-II* string theories [25]. But by a similar argument as presented here, those solutions should be the near horizon limits of Ep-branes in type-II* string theories and they are supersymmetric.

5. Conclusion

In this paper we have constructed and discussed various aspects of the static, non-supersymmetric p-brane (for $-1 \leq p \leq d-4$) solutions of type-II supergravities in diverse dimensions. We have solved the equations of motion of the relevant supergravity action by relaxing the supersymmetry (or extremality) condition by introducing a non-extremality function. Equations of motion dictate the three specific forms of the non-extremality function and hence the three specific classes of non-supersymmetric p-brane solutions. We have explicitly constructed all three classes of solutions. First two classes where the solutions are asymptotically flat are discussed in section 2, whereas the last class where the solutions are not asymptotically flat is discussed in section 4. We have also discussed the singularity structure and the region of validity of these solutions. Only one class of solutions discussed in section 2 were known before [4, 5] in a different form and we have clarified the relations of these solutions to those obtained here in section 2. As $p = -1$ or D-instanton solution is different from the other p-brane solutions, it is treated separately. Then we have discussed how a subclass of these solutions (with $G_-(r)$ as the non-extremality function) can be interpreted as the interpolating solutions between the p-brane–anti- p-brane system and the usual BPS p-branes in $d = 10$. In order to obtain BPS p-branes the parameters of the non-supersymmetric solutions were scaled in two different ways for $0 \leq p \leq 6$ and a unique way for $p = -1$. Then we also obtained BPS limits of the solutions with $G_+(r)$ as the non-extremality function. These give some unusual BPS brane solutions. We have also shown how the real time-dependent solutions or Sp-brane solutions (including S(-1)-brane) can be obtained from our static solutions by a Wick rotation. We have seen that this happens only for one class of solutions in section 2, but for the other class Wick rotation does not give real time-dependent solutions. For the third class of solutions, we found that although apparently the solutions are non-supersymmetric, however, by a coordinate transformation we have seen that they are nothing but the near-horizon limits of various BPS p-branes already known. We have given reasons why this happens for this particular class of solutions. Finally, we point out that the solutions we constructed in this paper have singularities. It would be interesting to understand the nature of the singularities and the ways, if at all possible, to resolve them.

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